Criticality Metrics

Lukas Westhofen et al.

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TIME-SCALE CRITICALITY METRICS

CHAPTER

ONE

INTRODUCTION

This page is supplementary material to the publication [Westhofen2022] and provides descriptions and evaluations of criticality metrics for automated vehicles.

The descriptions and assessments arose from a systematic suitability analysis, which is described in detail in the reference. It considers multiple properties that were deemed relevant for a broad albeit abstract set of applications the authors have observed criticality metrics being used in. Among others, we consider the metrics' target values, scenario types, specificities, and sensitivities. Definitions of the properties are given in the reference.

Note: Please note that when applying one or multiple of the described metrics in a practical, concrete setting, a detailed analysis of the specific application at hand is mandatory.

CHAPTER TWO

ABBREVIATIONS

For describing the criticality metrics, we use several abbreviations:

Abbreviation	Meaning
AV	Automated Vehicle
VRU	Vulnerable Road User
ODD	Operational Design Domain
ACC	Adaptive Cruise Control
AEB	Automated Emergency Braking
LKAS	Lane Keeping Assistance System
DMM	Dynamic Motion Model
MM	Maneuver Model
CA	Conflict Area
TT	Two Track
OT	One Track

CHAPTER

THREE

SYMBOLS

Within formulae, we use the following symbols:

Symbol	Meaning
A_i	actor i
$\mathcal{A}$	set of all actors in a scene or scenario
t_0	starting time of a scenario
t_e	ending time of a scenario
t	a point in time
t_H	a time horizon
$p_O(t)$	position of object O at time t
$p_i(t)$	position of actor i at time t
$p_{i,m}(t,t')$	position of actor i at time t when conducting maneuver m at time t'
$d(p_1(t), p_2(t))$	euclidean distance of $p_1(t)$ and $p_2(t)$
$\dot{d}(p_1(t), p_2(t))$	derivative of euclidean distance d
$v_i(t)$	velocity of actor i at time t
$a_i(t)$	acceleration of actor i at time t
$a_{i,min}(t)$	minimal available acceleration of actor i at time t
$a_{i,max}(t)$	maximal available acceleration of actor i at time t
$j_i(t)$	jerk of actor i at time t
ν_{long}	longitudinal component of a vector ν
ν_{lat}	lateral component of a vector ν
$u_i(t)$	control inputs of actor i at time t
$\beta_i(t)$	sideslip angle of actor i at time t
$\psi_i(t)$	yaw angle of actor i at time t
$\omega_i(t)$	yaw rate of actor i at time t
F_{idxy}	tire forces of actor i with direction d for tire (x, y)
$c_{i\alpha f}$	front tire cornering stiffness of actor i
$c_{i\alpha r}$	rear tire cornering stiffness of actor i
l_{if}	distance from front axle to center of gravity of actor i
l_{ir}	distance from rear axle to center of gravity of actor i
L	distance from front to rear axle
m_i	mass of actor i
I_{iz}	moment of inertia of actor i
δ_{if}	front steering angle at the tires of actor i
τ	target value
$\|\cdot\|_2$	the euclidean norm
ν_{long}	longitudinal component of a vector ν
ν_{lat}	lateral component of a vector ν

CHAPTER

FOUR

METRICS

Each criticality metric is described textually, accompanied by a formula. Furthermore, its properties are described concisely.

4.1 Encroachment Time (ET)

4.1.1 Description

The ET metric, proposed by Allen et al. [Allen1978], measures the time that an actor A_1 takes to encroach a designated conflict area CA, i.e.

$$ET(A_1, CA) = t_{\text{exit}}(A_1, CA) - t_{\text{entry}}(A_1, CA)\,.$$

While the value of ET is loosely correlated with criticality, it completely ignores the dynamics and behavior of any other involved actor.

4.1.2 Properties

Run-time capability

No

Target values

None found

Subject type

Road vehicles (automated and human)

Scenario type

Any scenario with a conflict area (containing a potential intersection point

Inputs

CA, $t_{entry}(A_1, CA)$, $t_{exit}(A_1, CA)$

Output scale

$[0, \infty)$, time (s), ratio scale

Reliability

Low, changes in criticality are reflected marginally in ET, repetition on multiple days showed high measurement variance [Allen1978]

Validity

Low, as only one actor is considered; validation against historical collision records was performed on one intersection, but results were not significant [Allen1978]

Sensitivity

Low, as the validity of the metric is low and no target values exist

Specificity

Low, as the validity of the metric is low and no target values exist

Prediction model

None, since a-posteriori

4.2 Post Encroachment Time (PET)

4.2.1 Description

The PET [Allen1978] has been widely used as metric for the a-posteriori analysis of traffic data [Laureshyn2010] [Peesapati2018] [Johnsson2018]. The PET calculates the time gap between one actor leaving and another actor entering a designated conflict area. Assuming A_1 leaves CA before or at the time A_2 enters it (i.e., $t_{\text{entry}}(A_2, CA) \geq t_{\text{exit}}(A_1, CA)$), we define

$$PET(A_1, A_2, CA) = t_{\text{entry}}(A_2, CA) - t_{\text{exit}}(A_1, CA)\,.$$

Note that the PET is undefined for scenarios where the above assumption does not hold. This can happen if both actors have entered CA before any of them was able to leave it. Moreover, depending on the definition of CA, a PET of 0 might not indicate an accident [Laureshyn2010].

Allen et al. also introduce two semi-predictive versions of the PET, called GT and IAPE, which inherit the properties of PET and are not considered any further here [Allen1978]. Both metrics, GT and IAPE, measure $t_{\text{exit}}(A_1, CA)$ and predict $t_{\text{entry}}(A_2, CA)$ at different points in time using a constant velocity model. Therefore, they can be seen as an evaluation of the PrET at a specific time point.

4.2.2 Properties

Run-time capability

No

Target values

<1 s [Varhelyi1998], 1.5 s [Peesapati2018] (threshold for critical), >2 s [Varhelyi1998] (normal interaction), multiple intersections, vehicle classes, and thresholds have been studied [Paul2020]

Subject type

Any two actors

Scenario type

Any scenario with a conflict area (containing a potential intersection point)

Inputs

CA, $t_{exit}(A_1, CA)$, $t_{entry}(A_2, CA)$

Output scale

$[0, \infty)$, time (s), ratio scale

Reliability

Can be reduced if reactive behavior leads to $t_{exit}(A_1, CA)$ or $t_{entry}(A_2, CA)$ being undefined (e.g. A_1 does full stop inside CA)

Validity

According to Allen, ‘conceptually sound descriptor’, correlation with collision history was highest, but non-significant [Allen1978], possibly reduced if actor bypasses CA for collision avoidance (e.g. emergency braking before reaching CA)

Sensitivity

Medium, as highly-critical braking maneuvers avoiding collisions may still result in a high PET due to re-accelerating [Zheng2019]

Specificity

High, as it is unlikely for an uncritical scenario to exhibit a low PET value due to the implicated spatio-temporal proximity of the actors

Prediction model

None, since a-posteriori

4.3 PTTC (Potential Time To Collision)

4.3.1 Description

The PTTC metric, as proposed by Wakabayashi et al. [Wakabayashi2003], constraints the general TTC metric by assuming constant velocity of A_1 and constant deceleration of A_2 in a car following scenario, where A_1 is following A_2. Then, the formula simplifies to

$$PTTC(A_1, A_2, t) = \frac{1}{-a_{2,long}(t)} \left(-\dot{d} \pm \sqrt{\dot{d}^2 + 2(-a_{2,long}(t))d} \right)$$

with $\dot{d} = \dot{d}(p_1(t), p_2(t))$ and $d = d(p_1(t), p_2(t))$ respectively. While imposing such constraints on the scenario type and the DMMs of the actors reduces the computational cost of evaluating the metric, its validity is significantly reduced compared to the general TTC.

4.3.2 Properties

Run-time capability

Yes

Target values

None found, but comparable to TTC

Subject type

Optimal for road vehicles (automated and human), sub-optimal for VRUs

Scenario type

Car following scenarios

Inputs

Positions p_i and velocities v_i for $i \in \{1, 2\}$ and acceleration a_2

Output scale

$[0, \infty]$, time (s), ratio scale

Reliability

Medium, but higher than a constant velocity TTC, as it increases the set of applicable scenarios by assuming constant deceleration of lead vehicle and constant velocity of following vehicle

Validity

Medium, as the assumption of constant deceleration of lead vehicle and constant velocity of following vehicle is not viable for all car following scenarios; was exemplarily demonstrated to be more valid than a constant velocity TTC [Wakabayashi2003]

Sensitivity

High inside the scenario type, as the metric's assumptions can be understood as an approximation of the worst-case for car following scenarios

Specificity

Low, as the assumptions may not be justified under all circumstances, thus metric can be raised too often

Prediction model

Time window

Bound by assumptions on acceleration

Time mode

Linear time

4.4 Predictive Encroachment Time (PrET)

4.4.1 Description

Here, we summarize the various predictive versions of the PET. The PrET [Neurohr2021] is the anticipated PET relative to an intersection point as predicted by the employed DMM, hence

$$PrET(A_1, A_2, t) = \min(\{|\tilde{t}_1 - \tilde{t}_2| \mid p_1(t + \tilde{t}_1) = p_2(t + \tilde{t}_2),\, \tilde{t}_1, \tilde{t}_2 \geq 0\} \cup \{\infty\})\,.$$

The Time Advantage (TA) metric [Hansson1975] can be interpreted as a special case of PrET for a constant velocity model, i.e. $p_i(s + t) = p_i(t) + sv_i(t)$. A scaled variant of the PrET, labeled Scaled Predictive Encroachment Time (SPrET), modifies the value of PrET by multiplication with the factor $(\tilde{t}_1 + \tilde{t}_2)$, i.e.

$$SPrET(A_1, A_2, t) = \min(\{|\tilde{t}_1^2 - \tilde{t}_2^2| \mid p_1(t + \tilde{t}_1) = p_2(t + \tilde{t}_2),\, \tilde{t}_1, \tilde{t}_2 \geq 0\} \cup \{\infty\})\,,$$

in order to decrease the weight of situations long before the predicted intersection [Neurohr2021]. Therefore, the SPrET incorporates prediction uncertainty.

4.4.2 Properties

Note: Includes properties for SPrET and TA.

Run-time capability

Yes

Target values

2 s (threshold for critical) and $[2, 3]$ s (normal traffic) for TA [Laureshyn2010], 3 s (threshold for critical) for SPrET [Neurohr2021]

Subject type

Any two actors

Scenario type

Any scenario with a conflict area (containing a potential intersection point)

Inputs

Static/dynamic objects and their state at time t, DMM for each object

Output scale

$[0, \infty]$, time (s), ratio scale

Reliability

Comparable to PET, but additionally dependent on DMM

Validity

Comparable to PET, but additionally dependent on DMM, increased validity for run-time applications; no empirical analysis available

Sensitivity

Comparable to PET, but additionally dependent on DMM

Specificity

Comparable to PET, but specificity decreases with increasing distance to intersection

Prediction model

Time window

Unbound, but usefulness depends on DMM

Time mode

Linear time

4.5 Time Exposed TTC (TET)

4.5.1 Description

The TET metric builds on the TTC together with a target value τ and is defined for a scenario as

$$TET(A_1, A_2, \tau) = \int_{t_0}^{t_e} \mathbf{1}_{TTC(A_1,A_2,t)\leq\tau} \mathrm{dt}$$

where $\mathbf{1}$ denotes the indicator function [Minderhoud2001] [Johnsson2018]. The TET measures the amount of time for which the TTC is below a given target value τ. The dependency of the TET on the scenario duration could easily be eliminated through division by $t_e - t_0$. Note that, while originally defined only for discrete time, we generalized the formula to continuous time. Moreover, let us mention that the idea of 'time exposed below target value' can readily be adapted to any metric together with a target value and is essentially independent of the TTC.

4.5.2 Properties

Run-time capability

Yes, but only retrospectively

Target values

None found

Subject type

Depends on the underlying metric, e.g.TTC

Scenario type

Depends on the underlying metric, e.g.TTC

Inputs

Inputs of the underlying metric, e.g.TTC, together with a target value τ

Output scale

$[0, t_e - t_0]$, time (s), ratio scale

Reliability

Medium, but greater than TTC alone due to reductions in fluctuations by integration over time

Validity

Depends on TTC, but at most medium, as a binary decision is made at each point in time, thus information is lost during aggregation. Can be valid for inter-scenario comparison [Minderhoud2001]

Sensitivity

Comparable to TTC, possibly reduced depending on τ

Specificity

Comparable to TTC, increased compared to using only TTC without time integration

Prediction model

Time window

Depends on the underlying metric

Time mode

Depends on the underlying metric, e.g. linear time in case of TTC

4.6 Time Headway (THW)

4.6.1 Description

The THW metric calculates the time until actor A_1 reaches the position of a lead vehicle A_2 [Jansson2005] [Junietz2018a].

$$THW(A_1, A_2, t) = \min\{\tilde{t} \geq 0 \mid p_1(t + \tilde{t}) = p_2(t)\}\,.$$

Analogously to THW, one can define the Headway (HW) metric [Jansson2005] simply as the distance to a lead vehicle, i.e.

$$HW(A_1, A_2, t) = d(p_1(t), p_2(t))\,.$$

4.6.2 Properties

Run-time capability

Yes

Target values

The THW, and HW in one example, are used by regulatory bodies in several countries to express driving recommendations and as a threshold for fines [Junietz2018a].

Country	Recommended	Threshold for fines
Germany	1.8 s	0.9 s
Sweden	3 s	1 s
Austria	2 s	0.4 s

Subject type

Road vehicles (automated and human)

Scenario type

Car following scenarios

Inputs

Static/dynamic objects and their state (pose, shape, etc.) at time t

Output scale

$(0, \infty]$, time (s), ratio scale

Reliability

Medium, comparable to HW with additional consideration of velocity

Validity

Medium, comparable to HW with additional consideration of velocity

Sensitivity

High, indicated by its use in legislation [Junietz2018a], also considers velocity compared to HW

Specificity

Low, as comparable to HW, but additional consideration of velocity

Prediction model

Time window

Unbound, but usefulness depends on DMM

Time mode

Linear time

4.7 Time Integrated TTC (TIT)

4.7.1 Description

Similar to the TET, the TIT [Minderhoud.2001] is a scenario level metric based on the TTC and is given as

$$TIT(A_1, A_2, \tau) = \int_{t_0}^{t_e} \mathbf{1}_{TTC(A_1,A_2,t)\leq\tau}(\tau - TTC(A_1, A_2, t))\mathrm{dt}.$$

It aggregates the difference between the TTC and a target value τ in the time interval $[t_0, t_e]$. Therefore, the metric reflects criticality more accurately than the TET. As for the TET, the construction of the TIT is independent of the TTC and can be adapted for other metrics.

4.7.2 Properties

Run-time capability

Yes, but only retrospectively

Target values

None found

Subject type

Depends on the underlying metric, e.g. TTC

Scenario type

Depends on the underlying metric, e.g. TTC

Inputs

Inputs of the underlying metric, e.g. TTC, together with a target value τ

Output scale

$[0, \infty)$, s · x, where x is unit of the underlying metric (e.g. time squared s^2 for TTC), ratio scale

Reliability

Medium, but greater than TTC alone due to reductions in fluctuations by integration over time

Validity

Higher than TET as continuous information is not lost during aggregation [Minderhoud2001], but also dependent on TTC

Sensitivity

Comparable to TTC, possibly even reduced depending on τ

Specificity

Comparable to TTC, increased compared to using only TTC without time integration [Guido2011]

Prediction model

Time window

Depends on the underlying metric

Time mode

Depends on the underlying metric, e.g. linear time in case of TTC

4.8 Time To Brake (TTB)

4.8.1 Description

Please refer to the *TTM*.

4.8.2 Properties

Run-time capability

Yes

Target values

0.4 s [Tamke2011] (selection of a minimal risk maneuver), 1 s [Junietz2018a] [Huber2020] (threshold for critical)

Subject type

Optimal for road vehicles (automated and human), sub-optimal for VRUs

Scenario type

Overlapping predicted trajectories for a significant time span in the scenario

Inputs

Static/dynamic objects and their state (pose, shape, etc.) at time t, MM for maneuver 'brake'

Output scale

$\{-\infty\} \cup [0, \infty]$, time (s), ratio scale

Reliability

High, under the assumption that collisions can be reliably predicted

Validity

High for two-actor scenes, medium for more actors when TTB is evaluated under a fixed perspective, but can be increased by aggregating over all actors as at least for one of them, the TTB will be valid

Sensitivity

High, but depends on the DMM: if no collisions are predicted for critical scenarios, sensitivity is reduced

Specificity

High for humans, as braking is a key choice in human reaction [Adams1994]; medium for AVs as non-braking maneuvers can often avoid a critical situation, even if TTB ≤ 0 [Adams1994]

Prediction model

Time window

Unbound, but usefulness depends on DMM

Time mode

Linear time

4.9 Time To Collision (TTC)

4.9.1 Description

For two actors A_1, A_2 at time t, the TTC metric returns the minimal time until A_1 and A_2 collide using an underlying one-track prediction model for both actors, or infinity if the predicted trajectories do not intersect. It is defined by

$$TTC(A_1, A_2, t) = \min\left(\{\tilde{t} \geq 0 \mid d(p_1(t+\tilde{t}), p_2(t+\tilde{t})) = 0\} \cup \{\infty\}\right).$$

A variety of the TTC, called Modified-TTC, is extended under the name of CrI, where it is multiplied with a velocity-based severity estimate [Ozbay2008].

For car following scenarios and from the point of view of a distinguished actor, the TTC delivers a quality estimate on the temporal proximity to a collision that is induced by a maneuver of an actors, e.g. by a braking maneuvers of a lead vehicle. Its validity is however greatly reduced for most DMMs within intersection scenarios as well as, if not meaningfully aggregated over actors, in multi actor scenes. Furthermore, the resulting time still needs to be interpreted w.r.t. the abilities and environment of A_1, either by using appropriate target values or composed metrics such as TTM.

One possible aggregate of the TTC to the scenario level is the TTA metric which is defined as

$$TTA(A_1, A_2) = TTC(A_1, A_2, t_{evasive})$$

with $t_{evasive}$ being the first time where an evasive maneuver is performed [Johnsson2018]. Such aggregations over time can increase the TTC's validity when used for a retrospective assessment. Further information is given when discussing the other two time aggregates of TTC in this work, TET and TIT.

4.9.2 Properties

Run-time capability

Yes

Target values

1 s [Hayward1972] [Huber2020], 1.5 s, [Sacchi2016], [ElBasyouny2013], 3 s [Autey2012] (all data separation), 1.22 s [Junietz2018a] (threshold for critical)

Subject type

Optimal for road vehicles (automated and human), sub-optimal for VRUs

Scenario type

Overlapping predicted trajectories for a significant time span in the scenario

Inputs

Static/dynamic objects and their state (pose, shape, etc.) at time t

Output scale

$[0, \infty]$, time (s), ratio scale

Reliability

Highly depending on the reliability of the predicted collision, for most DMMs reliability is reduced [Allen1978]

Validity

Medium, depending on the length of time interval with collision prediction in the scenario, as well as the validity of the DMM [StAubin2015]

Sensitivity

Medium, as, due to the linear-time DMM, critical scenes may not have a predicted collision in the DMM [Allen1978]

Specificity

High, as, due to the linear-time DMM, only few uncritical situations have a predicted collision in the DMM [Zheng2019]

Prediction model

Time window

Unbound, but usefulness depends on DMM

Time mode

Linear time

4.10 Time To Closest Encounter (TTCE)

4.10.1 Description

The TTCE is a distance-dependent risk indicator, which generalizes the concept of the TTC to the non-collision case [Eggert2014]. At time t, the TTCE measures the time $\tilde{t} > 0$ for which the distance to other actors in a scenario becomes minimal. The corresponding minimal distance is called the DCE. The formulae are

$$DCE(A_1, A_2, t) = \min_{\tilde{t} \geq 0} d(p_1(t + \tilde{t}), p_2(t + \tilde{t}))\,,$$

$$TTCE(A_1, A_2, t) = \text{arg}\min_{\tilde{t} \geq 0} d(p_1(t + \tilde{t}), p_2(t + \tilde{t}))\,.$$

In particular, as $DCE \to 0$, $TTCE \to TTC$ which implies $DCE = 0$ if and only if $TTCE = TTC$. Building on the TTCE and DCE, Eggert uses an exponential transform together with a survival function in order to estimate the future event probability of a collision for the distance-dependent risk [Eggert2014].

4.10.2 Properties

Run-time capability

Yes

Target values

None found

Subject type

Road vehicles (automated and human)

Scenario type

Any scenario

Inputs

Static/dynamic objects and their state (pose, shape, etc.) at time t

Output scale

$[0, \infty)$, time (s), ratio scale

Reliability

Higher than TTC as the DMM is not constraint to a predict a collision

Validity

Low, as the closest encounter is not necessarily a critical event, increased when used with a DCE threshold to delineate critical from non-critical encounters

Sensitivity

High, as many critical scenes exhibit temporal proximity to a close encounter

Specificity

Low, as a closest encounter is not always a critical event

Prediction model

Time window

Unbound, but usefulness depends on DMM

Time mode

Linear time

4.11 Time To Kickdown (TTK)

4.11.1 Description

Please refer to the *TTM*.

4.11.2 Properties

Run-time capability

Yes

Target values

None found

Subject type

Optimal for road vehicles (automated and human), sub-optimal for VRUs

Scenario type

Overlapping predicted trajectories for a significant time span in the scenario

Inputs

Static/dynamic objects and their state (pose, shape, etc.) at time t, MM for maneuver ‘kickdown’

Output scale

$\{-\infty\} \cup [0, \infty]$, time (s), ratio scale

Reliability

High, under the assumption that collisions can be reliably predicted

Validity

High if used in combination with other criticality metrics, unclear if used exclusively; no sources were found on the validity of examining kickdown maneuvers for threat mitigation

Sensitivity

Medium, due to kickdown not always being an actually realizable maneuver, thus high TTK values can be measured in effectively critical scenarios

Specificity

Medium, as potentially other feasible threat mitigation maneuvers exist (e.g. braking) even for scenes where TTK ≤ 0

Prediction model

Time window

Unbound, but usefulness depends on DMM

Time mode

Linear time

4.12 Time To Maneuver (TTM)

4.12.1 Description

The TTM metric returns the latest possible time in the interval $[0, TTC]$ such that a distinguished actor A_1 performing the considered avoidance maneuver would lead to collision avoidance with all other objects in the scene [Hillenbrand2006] [Tamke2011]. If $-\infty$ is returned, a collision cannot be avoided. Therefore,

$$TTM(A_1, A_2, t, m) = \max\left(\{\tilde{t} \in [0, TTC(A_1, A_2, t)] \mid d(p_{1,m}(t+s, t+\tilde{t}), p_2(t+s)) > 0 \,\forall\, s \geq \tilde{t}\} \cup \{-\infty\}\right).$$

The TTM can be extended to scenarios by aggregating over time and actors. For analytic purposes, an extension of the output scale to negative values is possible. Various special cases of the TTM metric have been considered [Tamke2011] [Wagner2018] [Junietz2018a], including Time To Brake (TTB) [Mages2009] (i.e., m = 'brake'), Time To Steer (TTS) [Hillenbrand2006] (i.e., m = 'steer'), and Time To Kickdown (TTK) [Hillenbrand2006] (i.e., m = 'kickdown').

4.12.2 Properties

Run-time capability

Yes

Target values

Depends on maneuver

Subject type

Optimal for road vehicles (automated and human), sub-optimal for VRUs

Scenario type

Overlapping predicted trajectories for a significant time span in the scenario

Inputs

Static/dynamic objects and their state (pose, shape, etc.) at time t, MM for the considered maneuver

Output scale

$\{-\infty\} \cup [0, \infty]$, time (s), ratio scale

Reliability

High, under the assumption that collisions can be reliably predicted

Validity

High, but depends on validity of the threat mitigation maneuver as well as the collision prediction of the DMM

Sensitivity

High, but depends on feasibility of the threat mitigation maneuver; if m is not feasible in actually critical scenarios, sensitivity is reduced

Specificity

High, but depends on maneuver m: if for a scene with $TTM \leq 0$ feasible threat mitigation maneuvers $\neq m$ exist, specificity is reduced

Prediction model

Time window

Unbound, but usefulness depends on DMM

Time mode

Linear time

4.13 Time To React (TTR)

4.13.1 Description

The TTR metric [Hillenbrand2006] [Tamke2011] approximates the latest time until a reaction is required by aggregating the maximum TTM metric over a predefined set of maneuvers M, i.e.

$$TTR(A_1, A_2, t) = \max_{m \in M} TTM(A_1, A_2, t, m)\,.$$

For example, as a set of maneuvers, one might select $M = \{$'brake', 'steer', 'kickdown'$\}$.

4.13.2 Properties

Run-time capability

Yes

Target values

Depends on maneuver set M

Subject type

Optimal for road vehicles (automated and human), sub-optimal for VRUs

Scenario type

Overlapping predicted trajectories for a significant time span in the scenario

Inputs

Static/dynamic objects and their state (pose, shape, etc.) at time t, set of maneuvers M and their MMs

Output scale

$\{-\infty\} \cup [0, \infty]$, time (s), ratio scale

Reliability

High, under the assumption that collisions can be reliably predicted

Validity

High, claimed to be 'an adequate metric to assess the criticality [...] since it directly relates to the driver's possible actions' [Hillenbrand2006], but also dependent on maneuvers and collision prediction

Sensitivity

Comparable to TTM, as, due to selection of the maneuver with the maximal TTM, the same reasoning applies

Specificity

Highly increased compared to TTM due to consideration of multiple maneuvers

Prediction model

Time window

Unbound, but usefulness depends on DMM

Time mode

Linear time for $|M| = 1$, branching time for $|M| > 1$

4.14 Time To Steer (TTS)

4.14.1 Description

Please refer to the *TTM*.

4.14.2 Properties

Run-time capability

Yes

Target values

0.47 s [Junietz2018a] (threshold for critical), 1 s [Huber2020] (ADF testing)

Subject type

Optimal for road vehicles (automated and human), sub-optimal for VRUs

Scenario type

Overlapping predicted trajectories for a significant time span in the scenario

Inputs

Static/dynamic objects and their state (pose, shape, etc.) at time t, MM for maneuver 'steer'

Output scale

$\{-\infty\} \cup [0, \infty]$, time (s), ratio scale

Reliability

High, under the assumption that collisions can be reliably predicted

Validity

High for two-actor scenes, medium for more actors when TTS is evaluated under a fixed perspective, but can be increased by aggregating over all actors as at least for one of them, the TTS will be valid

Sensitivity

High for non-humans, as steering is often optimal for threat mitigation than braking [Adams1994]; medium for humans as steering maneuvers are seldomly preferred [Adams1994]

Specificity

High under the assumption that the DMM has a low ratio of spurious collision among all predicted collision

Prediction model

Time window

unbound, but usefulness depends on DMM

Time mode

linear time

4.15 Time To Zebra (TTZ)

4.15.1 Description

Defined by Várhelyi et al., TTZ measures the time until an actor A_1 reaches a zebra crossing CA [Varhelyi1998], hence

$$TTZ(A_1, CA, t) = \min\left(\{\tilde{t} \geq 0 \mid d(p_1(t+\tilde{t}), p_{CA}(t+\tilde{t})) = 0\} \cup \{\infty\}\right).$$

Note that this concept can be further generalized to a Time To Object (TTO) metric for arbitrary moving or non-moving objects and conflict areas. For moving objects, this generalization coincides with the TTC.

4.15.2 Properties

Run-time capability

Yes

Target values

$[2, 4]$ s [Varhelyi1998] (critical interval at time of arrival of VRU)

Subject type

One road vehicle

Scenario type

Any scenario where a road vehicle is approaching a pedestrian crossing

Inputs

State of the approaching road vehicle, position of pedestrian crossing

Output scale

$[0, \infty]$‘, time (s), ratio scale

Reliability

Low, as changes in criticality are likely not reflected in TTZ, e.g. changes in walking directions of pedestrians

Validity

Low, as only the pedestrian crossing is regarded to be safety-relevant, and impact of other road users is not considered; no empirical analysis available

Sensitivity

Low, as VRUs can potentially cross in front of a pedestrian crossing, leading to critical scenarios with high TTZ values

Specificity

Low, if no VRU is present and the vehicle is close to the crossing, the scenario is uncritical, but TTZ approaches 0

Prediction model

Time window

Unbound, but usefulness depends on DMM

Time mode

linear time

4.16 Worst Time To Collision (WTTC)

4.16.1 Description

The WTTC metric extends the usual TTC by considering multiple traces of actors as predicted by an over-approximating DMM, i.e.

$$WTTC(A_1, A_2, t) = \min_{p_1 \in Tr_1(t), p_2 \in Tr_2(t)} (\{\tilde{t} \geq 0 \mid d(p_1(t+\tilde{t}), p_2(t+\tilde{t})) = 0\} \cup \{\infty\}),$$

where $Tr_1(t)$ resp. $Tr_2(t)$ denotes the set of all possible trajectories available to actor A_1 resp. A_2 at time t, as constraint by the employed DMM. Similar to the TTC, the WTTC can be extended to multi-actor scenarios. Defined by Wachenfeld et al. [Wachenfeld2016], it excels in selective data recording and data filtering applications.

4.16.2 Properties

Run-time capability

Yes

Target values

1 s (scenario classification) [Huber2020], comparison with ACC τ (gap time) made in [Wachenfeld2016]

Subject type

Optimal for road vehicles (automated and human), sub-optimal for VRUs

Scenario type

Overlapping predicted trajectories for a significant time span in the scenario

Inputs

Static/dynamic objects and their state (pose, shape, etc.) at time t

Output scale

$[0, \infty]$, time (s), ratio scale

Reliability

Medium, as over-approximating DMM robustly assesses criticality increases (expert-based evaluation [Wachenfeld2016]), but decreases potentially not reliably reflected

Validity

Medium, as most critical scenarios can be detected depending on the DMM, but also not able to distinguish many uncritical ones, initial expert-based evaluation on four scenarios has been published [Wachenfeld2016]

Sensitivity

Almost maximal, due to over-approximation of possible trajectories, depends on DMM (e.g. whether unstable dynamics are considered)

Specificity

Low, due to over-approximation of possible trajectories, depends on DMM

Prediction model

Time window

Unbound, but usefulness depends on DMM

Time mode

Branching time

4.17 Accepted Gap Size (AGS)

4.17.1 Description

The AGS is a quantity which can be used to measure the complexity of a traffic situation. In general it quantifies the gap or the actual space between actors desired or required for others to make a positive action decision. Therefore, for a given actor A_1 at time t, its model approximates the temporal or spatial distance that is predicted to be required for the action of A_1, i.e.

$$AGS(A_1, t) = \min\{s \geq 0 \,|\, action(A_1, t, s) = 1\},$$

where $action(A_1, t, s)$ is a (complex) model predicting on a binary scale, based on the circumstances at time t, whether A_1 will come to positive action decision for the gap size s.

This model can for example refer to the size of the gap in a stream of pedestrians passing a crosswalk, which is required for a waiting driver to decide to cut in and continue. For a time dependent distance measure, the metric is also called the accepted lag size. In general the more critical a traffic situation is, the larger the desired distance to other actors will be. For example, at an intersection, drivers tend to wait if the situation is unclear and the intersection itself is already crowded.

The interACT project used the AGS in such a way to analyze traffic complexity [InteractD61]. Their analysis shows that the accepted gap is a highly complex model depending on a vast set of inputs, such as gap size, driver age, gender, waiting time, distraction, and condition of the street. Furthermore, to the authors' knowledge, the formulation of the AGS has not yet been generalized and is bound to specific situations.

Alhajyaseen et al. studied the accepted gap resp. lag size for a left-turn traffic situation and empirically developed a probabilistic model using Cumulative Weibull distributions [Alhajyaseen2013]. The natural link between the accepted gap size and the Time To Arrival has been studied by Petzoldt [Petzold2014].

4.17.2 Properties

Run-time capability

Depends on model and inputs used

Target values

4.5s [Rakha2011] (for left-turn)

Subject type

Any, but requires a sound data basis or study for the gap acceptance model itself

Scenario type

Intersecting predicted paths for a significant time span in the scenario

Inputs

Static/dynamic objects and their state (pose, shape, etc.) at time t, other quantities for the underlying model (e.g. driver age or road condition)

Output scale

$[0, \infty)$, originally distance (m), sometimes seconds (s), ratio scale

Reliability

Depends on the influencing factors considered by $action$ model

Validity

High under the assumption of a valid data basis for the acceptance model w.r.t. the given scenario [Alhajyaseen2013]

Sensitivity

Medium, only aspects of criticality regarding gap acceptance are measured, thus critical scenarios may be missed (e.g. rear-end collisions)

Specificity

High, as an increased AGS is only present in highly complex situations which are often inherently critical

Prediction model

Time window

Depends on the quality of behavior prediction models

Time mode

Linear time

4.18 Distance of Closest Encounter (DCE)

4.18.1 Description

Please refer to the *TTCE*.

4.18.2 Properties

Run-time capability

Yes

Target values

None found

Subject type

Road vehicles (automated and human)

Scenario type

Any scenario

Inputs

Static/dynamic objects and their state (pose, shape, etc.) at time t

Output scale

$[0, \infty)$, distance (m), ratio scale

Reliability

Medium, often, changes in criticality will not be reflected in a change in spatial proximity, e.g. reductions of velocity

Validity

Medium, as the interpretation of the spatial proximity depends on other factors such as angles, velocities, and relative positioning; no empirical analysis available

Sensitivity

High, as many critical scenes exhibit a spatially close encounter

Specificity

Low, as a not all close encounters, specifically in unstructured or urban settings at low velocities, are directly critical

Prediction model

Time window

Unbound, but usefulness depends on DMM

Time mode

Linear time

4.19 Headway (HW)

4.19.1 Description

Please refer to the *THW*.

4.19.2 Properties

Run-time capability

Yes

Target values

50 m [Junietz2018a] (threshold for fines)

Subject type

Road vehicles (automated and human)

Scenario type

Car following scenarios

Inputs

Positions p_i for $i \in \{1, 2\}$, information to determine lead vehicles

Output scale

$[0, \infty)$, meter (m), ratio scale

Reliability

Low, as many changes in criticality (e.g. significantly higher speed in a situation) are not reflected by HW

Validity

Medium, a small distance to front does not always imply high criticality, but is also a factor present in many accidents

Sensitivity

High, as indicated by the use of HW in legislation [Junietz2018a], most critical situations do imply small distance to front for at least one involved actors

Specificity

Low, as a large distance to front does not always imply low criticality

Prediction model

Time window

Unbound, but usefulness depends on DMM

Time mode

Linear time

4.20 Proportion of Stopping Distance (PSD)

4.20.1 Description

The PSD metric, proposed by Allen et al., is defined as the distance to a conflict area CA divided by the MSD [Allen1978] [Mahmud2017] [Guido2011] [Astarita2012]. Therefore,

$$PSD(A_1, CA, t) = \frac{d(p_1(t), p_{CA}(t))}{\mathrm{MSD}(A_1, t)}, \mathrm{MSD}(A_1, t) = \frac{\|v_1(t)\|_2^2}{2|a_{1,long,min}(t)|} \,.$$

4.20.2 Properties

Run-time capability

Yes

Target values

< 1 (point of no return), 1.5 (scenario classification) [Huber2020]

Subject type

Road vehicles (automated and human)

Scenario type

Any scenario with a conflict area (containing a potential intersection point)

Inputs

CA, position p_1, velocity v_1, maximal deceleration of ego $a_{1,long,min}$

Output scale

$[0, \infty)$, number, ratio scale

Reliability

Can be reduced if actor tries to bypass conflict area in order to avoid collision, reliability higher than ET [Allen1978]

Validity

Low, depending on validity of the conflict area; found to have lowest relation to collision history for an unprotected left turn scenario with a static CA [Allen1978]

Sensitivity

High, assuming that the conflict area is defined as a dynamically changing predicted point of collision [Guido2011], reduced otherwise

Specificity

Low, if criticality is measured completely independent of other actors in the scenario, also low if defined relative to a predicted collision point [Guido2011]

Prediction model

Time window

Unbound, but usefulness depends on DMM

Time mode

Linear time

4.21 Conflict Severity (CS)

4.21.1 Description

CS is concerned with solely estimating the severity of a potential collision in a scenario [Bagdadi2013]. It thus presents as a suitable factor that can enhance various collision probability metrics in ensuring a more accurate representation of criticality. From the perspective of an actor A_1 performing a braking maneuver at time $t_{evasive}$, it is defined as

$$CS(A_1, A_2) = \Delta v(A_1, A_2, t_{evasive}) - \left(TTA(A_1, A_2) \cdot \|a_1(t_{evasive})\|_2 \cdot \frac{m_2}{m_1 + m_2} \right).$$

Thus, it compares the (extended) Δv at time of the evasive maneuver against the Δv at the potential collision point as predicted by TTA if A_1 conducts an emergency braking, assuming $v_2(t_{evasive} + TTA(A_1, A_2)) = 0$. CS factors in the relative mass difference due to the correlation between severe injuries and fatality outcome, measured on the Abbreviated Injury Scale, and the mass ratio of the involved actors [Evans1994].

4.21.2 Properties

Run-time capability

No, as TTA can only be computed once evasive maneuver has been identified

Target values

Case study on manually labeled critical scenarios identified a mean CS of 3.19 ± 3.7 m/s, but severity is dependent on speed and type of actors [Bagdadi2013]

Subject type

Any two actors, also suitable for VRUs

Scenario type

Any scenario for which a TTA can be determined

Inputs

$v_1(t_{evasive})$, $v_2(t_{evasive})$, m_1, m_2, $TTA(A_1, A_2)$, $a_{1,min}(t_{evasive})$

Output scale

$(-\infty, \infty)$, velocity (m/s), ratio scale

Reliability

Comparable to TTA

Validity

High validity for severity estimation inside scenario type of TTA, as indicated by empirical analysis against a traffic conflict technique, specifically for actors with different masses [Bagdadi2013]

Sensitivity

Medium, as for critical scenarios where no evasive maneuver is identified, CS returns no value

Specificity

Comparable to TTA, but increased due to consideration of severity

Prediction model

Depends on prediction model of TTA

4.22 Delta-v (Δv)

4.22.1 Description

Δv is the change in speed over collision duration and widely used in collision databases, where it is typically calculated from post-collision measurements [Gabauer2006]. Introduced in the 1970s [Carlson1979], it uses the difference in speed to estimate the probability of a severe injury or fatality:

$$\Delta v(A_1) = \|v_1(t_{aftercol}) - v_1(t_{beforecol})\|_2.$$

A more complex formula for two actors taking the masses into account is given by

$$\Delta v(A_1, A_2, t) = \frac{m_2}{m_1 + m_2}(\|v_2(t) - v_1(t)\|_2),$$

for which also probabilistic studies have been done [Shelby2011]. An extended Δv measure, which is additionally considering the mass as well as the driving angles of the collision participants, has been discussed by Laureshyn et al. [Laureshyn2017].

Joksch [Joksch1993] presents a model connecting Δv to the probability P of a two vehicle collision leading to a fatality using

$$P(A_1) \approx \left(\frac{\Delta v}{31.74\mathrm{m/s}}\right)^4.$$

This connection provides an easily interpretable measure.

4.22.2 Properties

Run-time capability

Yes

Target values

70 km/h [Shelby2011] (deadly collisions), 40 km/h [Ryb2007] (higher mortality rate)

Subject type

Any, but requires data basis for the target values depending on the involved traffic participants

Scenario type

Mainly collisions

Inputs

- Assuming one traffic participant: $v_1(t_{aftercol})$ and $v_1(t_{beforecol})$.
- Assuming two traffic participants: velocities v_i and masses m_i for $i \in \{1, 2\}$ over time

Output scale

$(-\infty, \infty)$, velocity (m/s), ratio scale, or $[0, 1]$, probability, ratio scale

Reliability

High reliability for collisions, greatly reduced for near-miss scenarios

Validity

High, if used to estimate injury severity of a collision [Gabauer2006]; very low if used for non-collision scenarios

Sensitivity

High for severity component of criticality, as it is often associated with a high Δv, might also depend on DMM and MM

Specificity

Medium, non-criticality of collisions can be partially evaluated, but other factors influence non-criticality, e.g. impact angle and torsional rigidity of the vehicles

Prediction model

None if used a-posteriori.

Otherwise:

Time window

Duration of collision

Time mode

Linear time

4.23 Deceleration to Safety Time (DST)

4.23.1 Description

For an actor A_1 following another actor A_2, the DST metric calculates the deceleration (i.e. negative acceleration) required by A_1 in order to maintain a safety time of $t_s \geq 0$ seconds under the assumption of constant velocity v_2 of actor A_2 [Hupfer1997] [Schubert2010]. The corresponding formula can be written as

$$DST(A_1, A_2, t, t_s) = \frac{(v_{1,long}(t) - v_{2,long}(t))^2}{2(d(p_1(t), p_2(t)) - v_{2,long}(t) \cdot t_s)}$$

and extends the concept of the $a_{long,req}$ by requiring deceleration to a safety distance $v_{2,long}(t) \cdot t_s$, under the assumptions of constant velocity of A_2, i.e. $a_2 = 0$. In particular, for $t_s = 0$, the DST agrees with the constant acceleration version of the $a_{long,req}$ metric.

4.23.2 Properties

Run-time capability

Yes

Target values

- < 1 m/s^2 (adaption)
- < 2 m/s^2 (reaction)
- < 4 m/s^2 (considerable reaction)
- < 6 m/s^2 (heavy reaction)
- ≥ 6 m/s^2 (emergency braking) for $t_s = 0$ [Hupfer1997]

Subject type

Road vehicles (automated and human)

Scenario type

Designed for car following, but can be extended to any scenario that potentially necessitates a braking maneuver

Inputs

Positions p_i and velocities v_i for $i \in \{1, 2\}$ and a safety time t_s

Output scale

$(-\infty, \infty)$, acceleration (m/s^2), ratio scale

Reliability

Comparable to $a_{long,req}$

Validity

Comparable to $a_{long,req}$, but depends on the validity of chosen value of t_s under the given circumstances, and assumption of constant velocity; was exemplarily shown to have improvements over TTC and PET [Hupfer1997]

Sensitivity

Comparable to $a_{long,req}$, large t_s increases sensitivity

Specificity

Comparable to $a_{long,req}$, large t_s decreases specificity

Prediction model

Time window

Limited, due to assumption of constant velocity

Time mode

Linear time

4.24 Required Lateral Acceleration ($a_{lat,req}$)

4.24.1 Description

Similar to the *required longitudinal acceleration*, the $a_{lat,req}$ [Jansson2005] is defined as the minimal absolute lateral acceleration in either direction that is required for a steering maneuver to evade collision. For two actors A_1, A_2 at time t, $a_{lat,req}$ measures the minimum absolute lateral acceleration required, on average, by actor A_1 to avoid a collision in the future:

$$a_{lat,req}(A_1, A_2, t) = \min\{|a_{1,lat}| \mid \forall\, \tilde{t} \geq 0 : d(p_1(t+\tilde{t}), p_2(t+\tilde{t})) > 0\}\,.$$

For actors A_1 and A_2 with constant acceleration where A_1 is following A_2, the formula concretizes to

$$a_{lat,req}(A_1, A_2, t) = \min\{|a_{1,lat,left}(A_1, A_2, t)|, |a_{1,lat,right}(A_1, A_2, t)|\}$$

where

$$a_{1,lat,k}(A_1, A_2, t) = a_{2,lat,k} + \frac{2(v_{2,lat}(t) - v_{1,lat}(t))}{TTC(A_1, A_2, t)} + \frac{2}{TTC(A_1, A_2, t)^2} \cdot \left[\pm\left(\frac{w_1 + w_2}{2}\right) + p_{2,lat}(t) - p_{1,lat}(t)\right]$$

with w_i denoting the width of A_i and $k \in \{left, right\}$ depends on the sign of $\frac{w_1+w_2}{2}$.

4.24.2 Properties

Run-time capability

Yes

Target values

$[-7, -2.5]$ m/s^2 dependent on speed [Benmimoun2011] (incident detection)

Subject type

Road vehicles (automated and human)

Scenario type

Intersecting predicted paths for a significant time span in the scenario

Inputs

v_i, a_i, p_i for $i \in \{1, 2\}$ assuming the constant acceleration motion model

Output scale

$(-\infty, \infty)$, acceleration (m/s^2), ratio scale

Reliability

High, under the assumption that collisions can be reliably predicted in the prediction model

Validity

High, but only lateral evasion considered, knowledge on vehicle capabilities necessary for interpretation [Zheng2019]

Sensitivity

High, as most critical situations between two actors impose a high required acceleration at some point

Specificity

Medium, as there exists situations with intersecting paths of actors, but planned trajectory is deviating (e.g. turning maneuvers)

Prediction model

Time window

Unbound, but usefulness depends on DMM

Time mode

Linear time

4.25 Required Longitudinal Acceleration ($a_{long,req}$)

4.25.1 Description

For two actors A_1, A_2 at time t, $a_{long,req}$ measures the maximum longitudinal backward acceleration required, on average, by actor A_1 to avoid a collision in the future. It can be formalized as

$$a_{long,req}(A_1, A_2, t) = \max\{a_{1,long} \leq 0 \mid \forall \tilde{t} \geq 0 : d(p_1(t+\tilde{t}), p_2(t+\tilde{t})) > 0\}\,.$$

The $a_{long,req}$ can be adapted for the situation where the acceleration of A_1 needs to be positive in order to avoid a collision by taking the minimum $a_{1,long} \geq 0$ instead. An interesting special case, cf. [Jansson2005], is exhibited when constant acceleration of the actors is assumed, resulting in

$$a_{long,req}(A_1, A_2, t) = \min\left(a_{2,long} + \frac{(v_{1,long}(t) - v_{2,long}(t))^2}{2d(p_1(t), p_2(t))}, 0\right).$$

For constant acceleration, the concept of $a_{long,req}$ is also known under the term Deceleration Rate To Avoid Crash (DRAC) [Archer2005]. Similarly, the $a_{lat,req}$ metric [Jansson2005] is defined as the minimal absolute lateral acceleration required for a steering maneuver to evade collision.

4.25.2 Properties

Run-time capability

Yes

Target values

-6 m/s^2 [Stellet2016] (AEB), $\mu \cdot g$ [Jeppsson2018] (point of no return), -3.4 m/s^2 [Huber2020] (scenario classification), $[-8, -4]$ m/s^2, dependent on speed [Benmimoun2011] (incident detection), -5 m/s^2 [UNECE157] (Requirement on emergency maneuver deployment in ALKS)

Subject type

Road vehicles (automated and human)

Scenario type

Intersecting predicted paths for a significant time span in the scenario

Inputs

v_i, a_i, p_i for $i \in \{1, 2\}$ assuming the constant acceleration motion model

Output scale

$(-\infty, \infty)$, acceleration (m/s^2), ratio scale

Reliability

High, under the assumption that the non-collision condition can be reliably predicted

Validity

High, but only longitudinal evasion considered, knowledge on vehicle capabilities necessary for interpretation [Zheng2019]

Sensitivity

High, as most critical situations between two actors impose a high required acceleration at some point, more sensitive than CPI [Guido2011]

Specificity

Medium, as there exists situations with intersecting paths of actors, but planned trajectory is deviating (e.g. turning maneuvers)

Prediction model

Time window

Unbound, but usefulness depends on DMM

Time mode

Linear time

4.26 Required Acceleration (a_{req})

4.26.1 Description

Based on $a_{long,req}$ and $a_{lat,req}$, the aggregate metric a_{req} can be defined in various ways [Jansson2005], e.g. by taking the norm of the required acceleration of both directions, i.e.

$$a_{req}(A_1, A_2, t) = \sqrt{a_{long,req}(A_1, A_2, t)^2 + a_{lat,req}(A_1, A_2, t)^2}\,.$$

More complex aggregates might also take into account the maximally available acceleration in each direction by incorporating the coefficient of friction μ. Also, let us mention the $a_{req,cond}$ [neurohr2021criticality] which combines a_{req} and SPrET for the analysis of urban intersection scenarios:

$$a_{req,cond}(A_1, A_2, t) = \begin{cases} a_{req}(A_1, A_2, t), \text{ if } SPrET(A_1, A_2, t) < 3s^2, \\ 0, \text{ otherwise.} \end{cases}$$

The $a_{req,cond}$ demonstrates by example how new criticality metrics can be created by combination of existing metrics and target values. In particular, the conditionality of the $a_{req,cond}$ encodes that the dynamical aspects of criticality only become relevant when a certain temporal criticality is present. This construction, of course, can be generalized as it is not specific to the a_{req} and SPrET. Generally, addressing the different aspects of criticality through combination of metrics could lead to vastly improved validity.

4.26.2 Properties

Run-time capability

Yes

Target values

-3.4 m/s^2 (scenario classification) [Huber2020], other values for lateral and longitudinal required acceleration may apply

Subject type

Road vehicles (automated and human)

Scenario type

Intersecting predicted paths for a significant time span in the scenario

Inputs

$a_{lat,req}$, $a_{long,req}$

Output scale

$(-\infty, \infty)$, acceleration (m/s^2), ratio scale

Reliability

High, under the assumption that the non-collision condition can be reliably predicted

Validity

High, found to be lower than TTC and PET for large thresholds [Zheng2019], but comparable to CPI [Guido2011]

Sensitivity

High, as most critical situations between two actors impose a high required acceleration at some point

Specificity

Medium, as there exists situations with intersecting paths of actors, but planned trajectory is deviating (e.g. turning maneuvers)

Prediction model

Time window

Unbound, but usefulness depends on DMM

Time mode

Linear time

4.27 Lateral Jerk (LatJ)

4.27.1 Description

Please refer to the *longitudinal jerk*.

4.27.2 Properties

Run-time capability

Yes

Target values

0.1 g/s (curve safety) [Ambros2019], 0.5 g/s [UNECE79], other upper bounds exist [ISO11270] [ISO22179]

Subject type

Road vehicles (automated and human)

Scenario type

Any involving a rapid steering maneuver, e.g. curve driving, evasion maneuver

Inputs

Jerk j_i at time t

Output scale

$(-\infty, \infty)$, jerk (g/s or m/s^3), ratio scale

Reliability

High, as jerk outcome is often in accordance with a change in criticality [Bagdadi2013]

Validity

Low for run-time applications, as jerk makes criticality only measurable on reaction, but high for a-posteriori analysis under the assumption of drivers being reacting to critical events [Bagdadi2013]

Sensitivity

High, since critical situations either require the driver to react abruptly, or the jerk will be high during the collision itself

Specificity

Low, as critical situations can happen without lateral acceleration of the actors

Prediction model

None, since instantaneous

4.28 Longitudinal Jerk (LongJ)

4.28.1 Description

Jerk is the rate of change in acceleration, and thus quantifies over the abruptness of a maneuver. The measure can simply be formulated as

$$LatJ(A_1, t) = j_{1,lat}(t), \quad LongJ(A_1, t) = j_{1,long}(t).$$

One of the main applications of the measure is the assessment of driving states. Using the jerk, it possible to discern different classes of driving styles, e.g. comfortable, angry, anxious, and risky modes [Bellem2018] [Feng2017]. Ambros et al. derived an indicator using the longitudinal jerk for the safety of a horizontal curve [Ambros2019]. Another important application area are trains and buses, where for standing passengers, the jerk enables an analysis of their reaction capabilities on the maneuver, e.g. during a change of tracks of a train [Powell2015].

The usage of LongJ and LatJ varies, e.g. LongJ can be utilized in the design of an ACC [ISO15622] function, whereas LatJ is used for functions dealing with steering maneuvers, e.g. an LKAS [ISO11270].

4.28.2 Properties

Run-time capability

Yes

Target values

1.2 g/s (reference driver) [UNECE157], 1.44 g/s (AEB) [UNECE157], other upper bounds exist [ISO11270] [ISO15622]

Subject type

Road vehicles (automated and human)

Scenario type

Any involving a rapid braking or kickdown maneuver

Inputs

Jerk j_i at time t

Output scale

$(-\infty, \infty)$, jerk (g/s or m/s^3), ratio scale

Reliability

High, as jerk outcome is often in accordance with a change in criticality [Bagdadi2013]

Validity

Low for run-time applications, as jerk makes criticality only measurable on reaction, but high for a-posteriori analysis under the assumption of drivers being reacting to critical events [Bagdadi2013]

Sensitivity

High, since critical situations either require the driver to react abruptly, or the jerk will be high during the collision itself

Specificity

Low, as critical situations can happen without longitudinal acceleration of the actors

Prediction model

None, since instantaneous

4.29 Aggregated Crash Index (ACI)

4.29.1 Description

The ACI measures the collision risk for car following scenarios by extending the concept of CPI from a_{req} to multiple conditions. First, a probabilistic causal model of the scenario type under consideration is needed to derive a collision tree with all possible outcomes and their probabilities [Kuang2015].

The concrete outcomes are represented by the tree's leaf nodes L_j. Every leaf node has a value C_{L_j} which is 0 in case of no collision and 1 in case of a collision. None-leaf nodes in the tree represent conditions which may occur during the scenario. Similar to CPI, the conditions are defined based on other metrics, e.g. the current stopping time of the lead vehicle being smaller than a lognormally distributed reaction time. The collision risk $CR_{L_j}(S)$ of a leaf node L_j given a scene S is hence represented by $CR_{L_j}(S) = P(L_j) \cdot C_{L_j}$, where $P(L_j)$ is the probability of satisfying all conditions necessary to reach L_j in the collision tree, when given the current conditions in the scene S.

The end result of ACI is an aggregation of all collision risks in a scene S, i.e.

$$ACI(S) = \sum_{j=1}^{n} CR_{L_j}(S),$$

with n being the number of leaf nodes in the collision tree.

4.29.2 Properties

Run-time capability

No

Target values

None found

Subject type

Road vehicles [Kuang2015], could be extended to other road users

Scenario type

Car following

Inputs

Static/dynamic objects and their state (pose, shape, etc.) at time t, probabilistic causal model and collision tree, probability for each scenario outcome

Output scale

$[0, 1]$, risk number, ratio scale

Reliability

Depends strongly on employed $a_mathit{min}$ model, reaction time model, and determination of probabilities

Validity

High, but depends on validity of conditions, empiric analysis shows improvements over TTC, PSD, and CPI [Kuang2015]

Sensitivity

Depends on how well the employed data set and collision tree represent the ground truth

Specificity

Depends on how well the employed data set and collision tree represent the ground truth

Prediction model

Time window

Depends on metrics used in conditions

Time mode

Branching time

4.30 Accident Metric (AM)

4.30.1 Description

AM evaluates whether an accident happened in a scenario:

$$AM(Sc) = \begin{cases} 0, & \text{no accident happened during } Sc, \\ 1, & \text{otherwise.} \end{cases}$$

This simplistic metric is implicitly used in accident databases, such as GIDAS. It fails to identify critical non-accident scenarios.

4.30.2 Properties

Run-time capability

No

Target values

Not necessary

Subject type

Any

Scenario type

Any

Inputs

Any parameter combination that allows determination of whether an accident happened or not

Output scale

$\{0, 1\}$, nominal scale

Reliability

Low, as a change in accident outcome can be related to only minor changes in criticality (i.e. near misses)

Validity

Medium, due to maximal specificity but low sensitivity

Sensitivity

Low, as all critical non-accidents are missed

Specificity

Maximal as all accidents are critical

Prediction model

None, since a-posteriori

4.31 Brake Threat Number (BTN)

4.31.1 Description

For actor A_1, the BTN [Jansson2005] is defined as the required longitudinal acceleration imposed on actor A_1 by actor A_2 at time t, divided by the longitudinal acceleration that is at most available to A_1 in that scene, i.e.

$$BTN(A_1, A_2, t) = \frac{a_{long,req}(A_1, A_2, t)}{a_{1,long,min}(t)} \, .$$

By definition, a BTN ≥ 1 indicates that a braking maneuver performed by the actor cannot avoid an impeding accident under the assumed DMM. An extension of BTN to multiple actors is proposed by Eidehall [Eidehall2011].

A special case of the BTN is known as the Deceleration-based Surrogate Safety Measure (DSSM). Here, for car-following scenarios, a worst case assumption of maximum braking of the lead vehicle is combined with an acceleration-dependent estimation of the following driver's time to perceive the threat and transition to emergency braking, thus incorporating human factors into the model [Tak2015].

4.31.2 Properties

Run-time capability

Yes

Target values

≥ 1 (point of no return)

Subject type

Road vehicles (automated and human)

Scenario type

Same as $a_{long,min}$

Inputs

$a_{long,req}$, $a_{long,min}$

Output scale

$(-\infty, \infty)$, number, ratio scale

Reliability

Comparable to $a_{long,req}$

Validity

Better than $a_{long,req}$ [Zheng2019], depends on $a_{long,req}$ and $a_{long,min}$ estimate; suited for inter-vehicle comparisons; no empirical analysis available

Sensitivity

High, but strongly depends on $a_{long,req}$ and direction of $a_{long,min}$ estimation

Specificity

High for humans, as braking is often preferred by human drivers [Adams1994]; strongly depends on $a_{long,req}$ and direction of $a_{long,min}$ estimation

Prediction model

Time window

Unbound, but usefulness depends on DMM

Time mode

Linear time

4.32 Conflict Index (CI)

4.32.1 Description

The conflict index enhances the acs{PET} metric with a collision probability estimation as well as a severity factor [Alhajyaseen2015]. For this, the estimated kinetic energy that would have been released assuming a hypothetical collision between A_1 and A_2 at their states when entering (A_2) resp. exiting (A_1) the conflict area is estimated:

$$CI(A_1, A_2, CA, \alpha, \beta) = \frac{\alpha \Delta K_e}{e^{\beta PET(A_1, A_2, CA)}}$$

where the denominator is a collision probability estimation.

Therefore, it is proposed that the actual collision probability is proportional to $e^{-\beta PET(A_1, A_2, CA)}$ with β being a calibration factor dependent on the scenario factors, e.g. country, road geometry, or visibility and $[\beta] = \mathrm{s}^{-1}$. The nominator represents a collision severity measure, where $\alpha \in [0, 1]$ is again a calibration factor for the proportion of energy that is transferred from the vehicle's body to its passengers and ΔK_e is the predicted absolute change in kinetic energy before and after the predicted collision.

ΔK_e is estimated based on the masses as well as velocities and angles at time of entering (A_2) resp. exiting (A_1) the conflict area.

4.32.2 Properties

Run-time capability

None, since PET can only be determined a-posteriori

Target values

None given

Subject type

Any two actors, but most suitable for road vehicles (automated and humans)

Scenario type

Any scenario with a conflict area (containing a potential intersection point)

Inputs

CA, $\theta_1(t_{exit}(A_1, CA)),\ \theta_2(t_{entry}(A_2, CA)),\ v_1(t_{exit}(A_1, CA)),\ v_2(t_{entry}(A_2, CA)),\ m_1, m_2,\ PET(A_1, A_2, CA)$, calibration factors α, β

Output scale

$(-\infty, \infty)$, joule (kg $\cdot$ m^2 $\cdot$ s^{-2}), ratio scale

Reliability

Comparable to PET

Validity

Initial validation was performed, exponential relationship to number of collisions over varying intersections was shown with a reasonably high coefficient of determination [Alhajyaseen2015]

Sensitivity

Depends on the sensitivity of PET, but potentially increased due to consideration of severity

Specificity

Depends on the specificity of PET, but potentially increased due to consideration of severity

Prediction model

None, since a-posteriori

4.33 Crash Potential Index (CPI)

4.33.1 Description

The CPI is a scenario-level metric and calculates the average probability that a vehicle can not avoid a collision by deceleration. It sums over the probabilities that a given vehicle's $a_{long,req}$ exceeds its $a_{long,min}$ for each time point and normalizes the value over the length of the scenario [Cunto2007] [Cunto2008]. The target value $a_{long,min}$ is assumed to be normally distributed and dependent on factors such as road surface material and vehicle brakes. While originally defined in discrete time, the CPI for a scenario can be defined in continuous time as

$$CPI(A_1, A_2) = \frac{1}{t_e - t_0} \int_{t_0}^{t_e} P(a_{long,req}(A_1, A_2, t) < a_{1,long,min}(t)) \mathrm{dt} \,.$$

Note that this concept of aggregation over time can be generalized to be applicable to other metrics, assuming that a valid probability distribution of the target value can be given. This potentially enables a more precise identification of criticality within a scenario.

4.33.2 Properties

Run-time capability

No

Target values

Average CPI was found to be 0.00491% in simulation, suggesting higher values as target values, e.g. 0.0072% (upper limit of 95%-confidence interval) [Cunto2008]

Subject type

Road vehicles (automated and human)

Scenario type

Intersecting predicted paths for a significant time span in the scenario

Inputs

$a_{long,req}$, $a_{long,min}$ probability distribution

Output scale

$[0, 1]$, probability, ratio scale

Reliability

Depends on reliability of $a_{long,req}$, but is potentially increased due to integration over time

Validity

Comparable to BTN, potentially increased due to comparison with a normally distributed target value, but depends on validity of distribution [Guido2011], initially validated [Cunto2008]

Sensitivity

Potentially high, but strongly depends on $a_{long,req}$ and validity of $a_{long,min}$ distribution for the given scenario

Specificity

Potentially high, but strongly depends on $a_{long,req}$ and validity of $a_{long,min}$ distribution for the given scenario

Prediction model

Time window

Unbound, but usefulness depends on DMM

Time mode

Linear time

4.34 Pedestrian Risk Index (PRI)

4.34.1 Description

The PRI estimates the conflict probability and severity for pedestrian crossing scenarios by combining the TTZ with the impact speed [Cafiso2011]. It is defined for a scenario with a vehicle A_1 and a VRU P both approaching a conflict area CA. The scenario shall include a unique and coherent conflict period $[t_{c_{start}}, t_{c_{stop}}]$ where $\forall\, t \in [t_{c_{start}}, t_{c_{stop}}]$: $TTZ(P, CA, t) < TTZ(A_1, CA, t) < t_s(A_1, t)$. Here, $t_s(A_1, t)$ is the time A_1 needs to come to a full stop at time t, including its reaction time, leading to

$$PRI(A_1, CA) = \int_{t_{c_{start}}}^{t_{c_{stop}}} (s_{imp}(A_1, CA, t)^2 \cdot (t_s(A_1, t) - TTZ(A_1, CA, t)))\mathrm{dt},$$

where s_{imp} is the predicted speed at the time of contact with the pedestrian crossing. The PRI thus quantifies over two aspects of a whole scenario: the temporal difference is claimed to be a surrogate for the accident probability, whereas the impact speed is approximate for its severity. One possibility of estimating s_{imp} is defined by the authors as

$$s_{imp}(A_1, CA, t) = \sqrt{\|v_1(t)\|_2^2 + 2a_{1,long,min}(t)(d(p_1(t), p_{CA}(t)) - \|v_1(t)\|_2 t_1^r)},$$

where t_i^r is the reaction time of actor A_i. Note that depending on the DMM, other formulae for s_{imp} may be employed.

4.34.2 Properties

Run-time capability

Theoretically possible, but primary design goal is a-posteriori analysis

Target values

Faded markings: 1992, visible markings: 1623, visible markings, speed bump: 407, raised visible markings, speed bump: 161 [Cafiso2011]

Subject type

Pedestrian

Scenario type

Scenarios with a unique and coherent conflict period, where one vehicle and pedestrian both approach a pedestrian crossing

Inputs

$a_{1,long,min}$, t_i^r, for each $t \in [t_{c_{start}}, t_{c_{stop}}]$: $v_i(t)$, $v_p(t)$, $t_s(A_i, t)$, $d(A_i, Z)$, $d(P, Z)$

Output scale

$(0, \infty)$, risk number (m^2/s^3), interval scale

Reliability

Depends on reliability of TTZ, but with the advantage of being robust against 'jumps' in TTZ due to integration over time

Validity

High if all assumptions hold, due to consideration of severity and probability, but dependent on validity of TTZ and v_{imp}; no empirical analysis available

Sensitivity

High, as constant velocity model can be considered an adverse estimation of future development at pedestrian crossings

Specificity

Medium, as prediction model does not consider reactive behavior of participants on each other

Prediction model

Same as TTA

4.35 Responsibility Sensitive Safety Dangerous Situation (RSS-DS)

4.35.1 Description

The Responsibility Sensitive Safety (RSS) framework is designed to formally guarantee safety during an automated vehicle's drive. It was developed to reflect a sound interpretation of law that leads to a efficient and verifiable AV behavior [Shalev-Shwartz2017]. To approach this goal, RSS states a set of mathematical rules.

For this, the safe lateral and longitudinal distances d_{min}^{lat} and d_{min}^{long} are formalized, depending on the current road geometry. The metric RSS-DS for the identification of a dangerous situation S with a set of actors $\mathcal{A}$ is defined as

$$RSS\text{-}DS(A_1, \mathcal{A}) = \begin{cases} 1, \exists A_i \in \mathcal{A} \setminus \{A_1\}. d^{lat}(A_1, A_i) < d_{min}^{lat} \wedge d^{long}(A_1, A_i) < d_{min}^{long}, \\ 0, \text{otherwise.} \end{cases}$$

Note that to determine d_{min}^{lat} and d_{min}^{long}, different prediction models are utilized to estimate which distances are classified as safe, e.g. for intersections, highways, and unstructured roads.

Note that RSS has been shown to not consider certain edge cases, e.g. during braking maneuvers and on varying road surfaces and slopes, as well as the issue of perception uncertainty [Koopman2019].

An extension of RSS-DS measures the temporal extent to which the ego was not able to mitigate the dangerous situation [Jesenski2020]. In accident research, a similar concept of classifying situations as safe and unsafe depending on longitudinal stopping distances was introduced as the Stopping Distance Index (SDI) [Oh2006]. In turn, the SDI is partially based on the idea of the Potential Index for Collision with Urgen Deceleration (PICUD) [Uno2002], both comparing the stopping distances of the lead and following vehicle under emergency braking.

4.35.2 Properties

Run-time capability

Yes

Target values

Not necessary

Subject type

Road vehicles (esp. suitable for automated vehicles, but also possible to evaluate human drivers)

Scenario type

RSS ODD (also suited for urban, unstructured scenarios)

Inputs

$d^{lat}(A_1, A_i)$ and $d^{long}(A_1, A_i)$ for all $A_1 \neq A_i$, response time ρ and other inputs required to predict d^{lat}_{min} and d^{long}_{min}

Output scale

$\{0, 1\}$, number, nominal scale

Reliability

Medium, the nominal nature of the metric's scale can lead to fluctuations if vehicles exist close the boundaries of the safe distancesMedium, the nominal nature of the metric's scale can lead to fluctuations if vehicles exist close the boundaries of the safe distances

Validity

High, depending mainly on the validity of the safe distance definition of the scenario (e.g. highways or unstructured roads) [Chai2019]

Sensitivity

High, due to over-approximation of safe space [Chai2019], although reduced for edge cases [Koopman2019]

Specificity

Medium, as not every violation of a safe space directly implies high criticality [Chai2019], but depends on the definition of the safe distances

Prediction model

Time window

Depends on model for d_{min} prediction, for single lane roads $\rho + TTB$

Time mode

Linear time

4.36 Space Occupancy Index (SOI)

4.36.1 Description

The SOI defines a personal space for each actor and counts violations by other participants while setting them in relation to the analyzed period of time [Tsukaguchi1987] [Ogawa2007] [Johnsson2018]. For each actor A_i at time t, a personal space $Sp(A_i, t)$ is defined. At time t, if there exists some $A_j \neq A_i$ s.t. $Sp(A_i, t) \cap Sp(A_j, t) \neq \emptyset$, a violation of the personal space of A_i is given. The number of conflicts is then given as $C(A_1, \mathcal{A}, t) = \sum_{A_j \in \mathcal{A} \setminus \{A_1\}} [Sp(A_1) \cap Sp(A_j) \neq \emptyset]$, where $[\cdot]$ denotes the Iverson bracket. Thus, for a given scenario in the time interval $[t_0, t_e]$, the conflict index is defined as

$$SOI(A_1, \mathcal{A}) = \sum_{t=t_0}^{t_e} C(A_1, \mathcal{A}, t).$$

SOI was introduced for bicycles and pedestrians, however, it is possible to formulate a similar concept for road vehicles.

4.36.2 Properties

Run-time capability

Yes, but only retrospectively

Target values

No

Subject type

Originally defined for VRUs, could be extended to road vehicles

Scenario type

Any scenario

Inputs

Actor type, size of safe space depending on type

Output scale

$[0, \infty)$, hertz (1/s), ratio scale

Reliability

Medium, due to the nominal nature of the conflict definition, which leads to fluctuations if vehicles exist close the boundaries of personal space

Validity

Medium, since temporal and dynamical aspects are ignored due to binary evaluation; no empirical analysis available

Sensitivity

Medium, as already a single safe space violation can lead to an accident

Specificity

Medium, as multiple safe space violations are associated but with but not necessarily causative for accidents

Prediction model

None, since a-posteriori

4.37 Steer Threat Number (STN)

4.37.1 Description

Similar to the BTN, for two actors at time t, the STN [Jansson2005] [Eidehall2011] is defined as the required lateral acceleration divided by the lateral acceleration that is at most available to A_1 in that direction in that scene, i.e.

$$STN(A_1, A_2, t) = \frac{a_{lat,req}(A_1, A_2, t)}{a_{1,lat,min}(t)}.$$

By definition, an STN ≥ 1 indicates that a lateral maneuver performed by the actor cannot avoid an impeding accident.

4.37.2 Properties

Run-time capability

Yes

Target values

≥ 1 (point of no return)

Subject type

Road vehicles (automated and human)

Scenario type

Same as $a_{lat,req}$

Inputs

$a_{lat,req}$, $a_{lat,min}$

Output scale

$(-\infty, \infty)$, number, ratio scale

Reliability

High, under the assumption that the non-collision condition can be reliably predicted

Validity

Strongly depends on $a_{lat,req}$ and $a_{\text{lat,min}}$ estimate; suited for inter-vehicle comparisons; no empirical analysis available

Sensitivity

High for non-humans, as steering is often optimal, but seldomly executed by humans [Adams1994]; strongly depends on $a_{lat,req}$ and direction of $a_{lat,min}$ estimation

Specificity

High, but strongly depends on $a_{lat,req}$ and direction of $a_{lat,min}$ estimation

Prediction model

Time window

Unbound, but usefulness depends on DMM

Time mode

Linear time

4.38 Trajectory Criticality Index (TCI)

4.38.1 Description

The ac{TCI} metric models criticality using an optimization problem [Junietz2018]. The task is to find a minimum difficulty value, i.e. how demanding even the easiest option for the vehicle will be under a set of physical and regulatory constraints. For example, if the constraint is to avoid obstacles, then driving straight towards an obstacle and being only a few seconds away requires a large change in steering angle and acceleration to satisfy the constraint of collision avoidance.

Here, the possible set of vehicle actions are not only constrained by physically possible behavior; it additionally shall adhere to a mathematically modeled set of requirements. Said requirements are based on the necessary longitudinal (a_x) and lateral acceleration (a_y) to avoid collisions as well as the margin ('reserve') for corrections in speed (R_x) and course angle (R_y). Since both R_x and R_y are dependent on a_x and a_y, it suffices to minimize the combined function w.r.t. a_x and a_y. The requirements include concepts such as holding a safe following distance and maximizing distance to obstacles.

Assuming the vehicle behaves according to Kamm's circle, acs{TCI} for a scene S with an ego vehicle A_1 reads as .. math:

```
\mathit{TCI}(A_1,S,t,t_H) = \min_{a_x, a_y} \sum_{\tilde{t}=t}^{t+t_H} w_x R_x(\tilde{t}
↪) + w_y R_y^2(\tilde{t}) + \frac{w_{\mathit{ax}} a_x^2(\tilde{t}) + w_{\mathit{ay}} a_
↪y^2(\tilde{t})}{(\mu_\mathit{max}g)^2}
```

where t_H is the prediction horizon, a_x and a_y the longitudinal and lateral accelerations, μ_{max} the maximum coefficient of friction, g the gravitational constant, w weights, and R_x and R_y the longitudinal and lateral margin for angle corrections:

$$R_x(t) = \frac{\max(0, x(t) - r_x(t))}{d_x(t)},$$
$$R_y^2(t) = \frac{(y(t) - r_y(t))^2 v(t - t_s)}{d_y^2(t) v_{max}}.$$

Here, $x(t)$, $y(t)$ is the position, t_s the discrete time step size, v_{max} the maximum velocity, $r_x(t)$ the reference for a following distance (set to $2\mathrm{s} \cdot v(t)$), r_y the position with the maximum lateral distance to all obstacles in S, $d_x(t)$, $d_y(t)$ the maximum longitudinal and lateral deviations from r_x, r_y.

4.38.2 Properties

Run-time capability

Yes, but not designed for active trajectory control

Target values

None found

Subject type

Road vehicles (esp. automated)

Scenario type

Mostly highways

Inputs

Velocities v_i, positions p_i, following distance r_x, position maximizing lateral distances r_y, μ_{max} maximum coefficient of friction

Output scale

$[0, \infty]$, number, ratio scale

Reliability

High, under the assumption of a reliable encoding of safety-critical factors in constraints

Validity

High, as demonstrated by an expert-based assessment of the metric's results in four scenarios, but also found to be dependent on the cost function and constraints [Junietz2018]

Sensitivity

High, no false negative identified in initial expert-based validation [Junietz2018], but also depends on cost function and constraints

Specificity

High, no false positive identified in initial expert-based validation [Junietz2018], but also depends on cost function and constraints

Prediction model

Time window

Unbound, but depends on computational power and choice of cost function and constraints

Time mode

Branching time

4.39 Collision Probability via Monte Carlo (P-MC)

4.39.1 Description

P-MC produces a collision probability estimation based on future evolutions from a Monte Carlo path planning prediction [Broadhurst2005]. At first, a binary representation of the road geometry with the distinction of drivable and non-drivable is generated. If the ego enters a non-drivable region, a collision is detected. Every object in the scene has a state, denoted by $s_i(t) = (p_i(t), v_i(t))$, and control inputs $u_i(t)$. The motion of each object is then described by an ODE of the form $\dot{s}_i(t) = f(s_i(t), u_i(t))$. For example stationary obstacles are modeled by $f \equiv 0$ and vehicles are modeled by the simple car state update equation

$$\begin{pmatrix} \dot{s}^{(1)} \\ \dot{s}^{(2)} \\ \dot{s}^{(3)} \\ \dot{s}^{(4)} \end{pmatrix} = \begin{pmatrix} s^{(3)} \cos(s^{(4)}) \\ s^{(3)} \sin(s^{(4)}) \\ u^{(1)} \\ \frac{s^{(3)}}{L} \sin(u^{(3)}) \end{pmatrix}.$$

If the bounding boxes of two objects intersect at some point between t and $t+t_H$, a collision is detected. A goal function $g(u_i(t))$ is defined for each object in the scene to specify the desirability of paths that the object might follow based on the possible control inputs. Various choices for this goal function can be made, influencing the prior distribution P. With k objects in a scene, the combined goal of all objects is defined as

$$P(\mathcal{U}) := P(u_1, \dots, u_k) := \prod_{j=1}^{k} P(u_j)^{\alpha_j}$$

For an actor A_1 in a scene S, the collision probability is then

$$P\text{-}MC(A_1, S, t) = P(\mathcal{C}) = \int P(\mathcal{C} \mid \mathcal{U}) P(\mathcal{U}) d\mathcal{U} \ ,$$

with $P(\mathcal{C} \mid \mathcal{U})$ being the collision probability of A_1 in S under the given inputs $\mathcal{U}$.

4.39.2 Properties

Run-time capability

Yes

Target values

None found

Subject type

Any, but requires behavior and dynamic model of subject

Scenario type

Depends on the validity of the models

Inputs

Static/dynamic objects and their state, estimated bounding boxes, possible control inputs, behavior model for each object

Output scale

$[0, 1]$, probability, ratio scale

Reliability

High, but this largely depends on the models used and thus also on available computational power

Validity

High, but this largely depends on the models used and thus also on available computational power; no empirical analysis available

Sensitivity

High, but this largely depends on the models used and thus also on available computational power

Specificity

High, but this largely depends on the models used and thus also on available computational power

Prediction model

Time window

Depends on the computational power, used behavior and MM and complexity of the situation

Time mode

Branching time

4.40 Collision Probability via Scoring Multiple Hypotheses (P-SMH)

4.40.1 Description

Similar to other probability-based approaches, Sánchez Morales et al. propose to assign probabilities to predicted trajectories and accumulate them into a collision probability [Morales2019]. The motion of the ego is modeled by a two track model. Due to less information being known with a reasonable accuracy for the other actors, a one track model is used for those. Pedestrians have the ability of changing direction, velocity, and acceleration in a finite set of steps under given constraints. Once the number N of trajectories for the ego and total number M of trajectories of all other actors is determined, one can compute the collision probability as

$$P\text{-}SMH(A_1, \mathcal{A}, t) = \sum_{i=1}^{N} \sum_{j=1}^{M} \chi_j^i p_{A_1,i} p_{(\mathcal{A} \setminus A_1),j} \ ,$$

where χ_j^i equals one if and only if the i-th trajectory of A_1 and the j-th trajectory of the actors in $\mathcal{A} \setminus A_1$ lead to a collision, and $p_{A_1,i}$ resp. $p_{(\mathcal{A} \setminus A_1),j}$ are the probabilities of the trajectories being realized.

4.40.2 Properties

Run-time capability

Yes, demonstrated by evaluation [Morales2019]

Target values

None found

Subject type

Any, but requires behavior and dynamic model of subject

Scenario type

Depends on definition of models

Inputs

Static and dynamic objects as well as their state, estimated bounding boxes, ego: see TT model, other vehicles: see OT model

Output scale

$[0, 1]$, probability, ratio scale

Reliability

High, as the consideration of multiple futures and their likelihoods makes it robustly follow changes in criticality [Morales2019]

Validity

High, due to branching predictions and likelihood estimation, but depends on the validity of the motion model and probabilities, initial simulative validation results exist [Morales2019]

Sensitivity

High, but depends on the validity of the motion model and available computational power, no analysis of false negatives was performed in initial evaluation [Morales2019]

Specificity

High, an initial evaluation found no false positives by the metric [Morales2019]

Prediction model

Time window

Unbound, but longer prediction horizons at a constant number of predicted trajectories lower reliability

Time mode

Branching time

4.41 Collision Probability via Stochastic Reachable Sets (P-SRS)

4.41.1 Description

Althoff et al. propose to estimate a collision probability using stochastic reachable sets [Althoff2009]. Firstly, the reachable set $R([t, t+t_H])$ (the set of possible positions until the horizon t_H) is over-approximated for each actor, where the movement of the actor is approximated by Markov chains with time steps $\{t+t_1, t+t_2, \ldots, t+t_H\}$ and a constant $T = t_{k+1} - t_k$. Due to computational effort, the abstraction from continuous models to Markov chains has to be pre-computed offline for real-time execution of the metric. The ego's motion is not modeled as it is assumed to be known.

Afterwards, the state and input space are discretized, thus we can write $R_i^\alpha(T)$ for the reachable set given a state in the i-th partition of the state space and the input in the α-th partition of the input space for time T. The transition probabilities to partitions X_j of the state space are given by

$$\Phi_{ji}^\alpha(T) = \frac{V(R_i^\alpha(T) \cap X_j)}{V(R_i^\alpha(T))}$$

where V returns the volume. Aforementioned concepts are then generalized to $\Phi_{ji}^\alpha([0,T])$ by substituting $R_i^\alpha(T)$ with

$$R_i^\alpha([0,T]) = \bigcup_{t \in [0,T]} R_i^\alpha(t)$$

not accounting for the discrete time aspect at this point [Althoff2009]. The transition probabilities can then be used to obtain the probability distribution for the time intervals by

$$p(t_{k+1}) = \Phi^\alpha(T) \cdot p(t_k)$$

$$p([t_k, t_{k+1}]) = \Phi^\alpha([0,T]) \cdot p(t_k)$$

again simplified for readability. Behaviors of other actor are modeled as Markov chains on the control input space of the motion models. Due to the discretization of the state space, we can approximate the lateral deviation by a piecewise constant function and thus we can define intervals D_f where said function is constant. This leads to a lateral position probability of

$$p_f^{dev}([t_k, t_{k+1}]) = P(\delta \in D_f, t \in [t_k, t_{k+1}]).$$

By splitting the state space partitions X_i into position and velocity, i.e. $X_i = S_e \times V_m$, one can define

$$p_e^{path}([t_k, t_{k+1}]) = \sum_m P(s \in S_e, v \in V_m, t \in [t_k, t_{k+1}]).$$

Afterwards, all possible paths in which two actors could have intersecting vehicle bodies are identified and stored in a list Ω. This list is finite due to the piecewise constant partitions. Under the assumption of stochastic independence and using the previous concepts, we then have $p_{ef}^{pos} = p_e^{path} \cdot p_f^{dev}$, hence leading to the collision probability

$$P\text{-}SRS(A_1, S, t) = p^{col} = \sum_{(g,h,e,f)\in\Omega} \hat{p}_{gh}^{pos} \cdot p_{ef}^{pos}.$$

4.41.2 Properties

Run-time capability

Yes, with precomputation

Target values

None found

Subject type

Any, but requires behavior and dynamic model of subject

Scenario type

Any scenario, depends on the validity of the models

Inputs

Static/dynamic objects and their state, estimated bounding boxes, possible control inputs, behavior models, various constants of objects in the scene

Output scale

$[0, 1]$, probability, ratio scale

Reliability

High, as the consideration of multiple futures and their likelihoods makes it robustly follow changes in criticality

Validity

High, but largely depends on the models, available computational power and discretization coarseness [Althoff2009], no representative empirical analysis found

Sensitivity

High, but largely depends on the models and available computational power

Specificity

High, but largely depends on the models and available computational power

Prediction model

Time window

Unbound, but largely depends on available computational power

Time mode

Branching time

4.42 Potential Functions as Superposition of Scoring Functions (PF)

4.42.1 Description

The general concept of the PF metric is to define a potential function for each static or dynamic object considered by the metric [Wolf2018]. This includes potentials for lane markings, the road geometry, other vehicles, or, in more urban areas, pedestrians and bicyclists. Once a potential function for each object in the scene, denoted by $U_i(A, S)$, is chosen, one can apply e.g. gradient descent for a given scene S to the combined potential function $U(A, S) = U_1(A, S) + \cdots + U_k(A, S)$, where A is an actor and k denotes the number of objects. A simple example of how to evaluate this metric for an actor A_1 and a given scene S' is by inserting the values into U, i.e.

$$PF(A_1, S') = U(A_1, S') = U_1(A_1, S') + \cdots + U_k(A_1, S') \,.$$

However, methods involving the mentioned gradient descent to assess the criticality can improve precision and also provide a suggestion for criticality-reducing vehicle movement.

Due to the way this metric is defined, almost all properties depend on the specified potential functions. Furthermore, while ethical questions play a role when defining any safety surrogate, it becomes more evident for potential functions, as an active decision making in the definition of the potentials is required.

4.42.2 Properties

Run-time capability

Yes

Target values

None found, also highly dependent on the used potential functions

Subject type

Any, but requires a potential function for each considered subject type

Scenario type

Depends on specified potential functions

Inputs

Potential function for each static/dynamic object in the scene that is supposed to be considered, other inputs depend entirely on said potential functions

Output scale

$[-\infty, \infty]$, number (negative values are possible if goal locations are defined), ratio scale

Reliability

Largely depends on the used potential functions

Validity

Largely depends on the used potential functions; no empirical analysis identified

Sensitivity

Largely depends on the used potential functions

Specificity

Largely depends on the used potential functions

Prediction model

Time window

Depends on quality of potential functions and reliability of computation of the solution to the potential equation problem

Time mode

Branching time

4.43 Safety Potential (SP)

4.43.1 Description

The SP is a part of the Safety Force Field (SFF) framework, which proposes a method to compute safe control policies on a collision-avoidance level. Conceptually, the SFF tries to identify, under the assumption of all actors conducting some safe control policy (e.g. an emergency brake), whether there can exist a conflict [Nister2019]. To measure how unsafe w.r.t to collision avoidance a situation is, SFF uses SP as a numeric valuation.

SFF assumes that each actor $A_1 \in \mathcal{A}$ has a set of safe control policies, S_1. Each safe control policy $s \in S_1$ brings an actor A_1 to a full stop in finite time. SFF defines the occupied set O_1 of an actor A_1 to include its safety margin as well as A_1 itself. For each point on each trajectory that can arise from conducting a safe control policy $s \in S_1$, O_1 is examined. The resulting union of trajectories is the claimed set C_1.

The unsafe set between two actors $A_1, A_2 \in \mathcal{A}$ can then be identified as $U_{1,2} = \{x \in C_1 \times C_2 \mid C_1(x) \cap C_2(x) \neq \emptyset\}$. Intuitively, it is the set of all actor state combinations for which there exist safe control policies leading to a collision.

Identifying the combined state space of A_1 and A_2 as $\Omega_1 \times \Omega_2$, SFF subsequently employs a potential function $\rho_{1,2} : \Omega_1 \times \Omega_2 \to \mathbb{R}$ to rate the combined states of actors, where

- $\rho_{1,2}(u) > 0$ for all $u \in U_{1,2}$ and
- $\rho_{1,2}(u) \geq 0$ for all $u \notin U_{1,2}$ and
- $\rho_{1,2}(x) \geq \rho_{1,2}(x')$ if x' is a state derived from x by A_1, A_2 applying $s_1, s_2 \in S_1, S_2$.

The safety potential can hence rate a two-actor scene from one of their perspectives.

The authors state the following exemplary safety potential for some $k \in \mathbb{Z}_{>0} \cup \{\infty\}$:

$$SP(A_1, A_2, t) = \rho_{1,2} = \|(t_{stop}(A_1) - t_{int}, t_{stop}(A_2) - t_{int})\|_k$$

where t_{int} is the the earliest intersection time when continuing the current situation under some model, and $t_{stop}(A_i)$ is the time of full stop of A_i after applying a safety procedure.

Note that this framework can be extended with various more complex safety potentials [Nister2019]. Downstream, SFF uses the gradient of the safety potential to optimize for a safe control policy, if necessary.

4.43.2 Properties

Run-time capability

Yes

Target values

No

Subject type

Automated road vehicles

Scenario type

Any for whose entities corresponding safety potentials and procedures can be defined

Inputs

For k actors: states (e.g. p_i, d_i, v_i), safety procedures S_i and definition of safety potential $\rho_{i,j}$ for $i, j \in \{1, \ldots, k\}$

Output scale

$[0, \infty)$, number, ordinal scale

Reliability

High, but additionally depends on the reliability of the safety potentials

Validity

High inside time window, but greatly dependent on validity of potential definition; no empirical analysis available

Sensitivity

Potentially high, but depends on safety procedures and potential definition

Specificity

Potentially high, but depends on safety procedures and potential definition

Prediction model

Time window

Duration of safety procedure

Time mode

Branching time

4.44 References

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